Assignment 12

This homework is due Wednesday April 27.

There are total 34 points in this assignment. 30 points is considered 100%. If you go over 30 points, you will get over 100% for this homework (but not over 115%) and it will count towards your course grade.

Collaboration is welcome. If you do collaborate, make sure to write/type your own paper *and give credit to your collaborators in your pledge.* Your solutions should exhibit your work and contain full proofs. Bare answers will not earn you much.

This assignment covers Sections 7.3–7.5 of Textbook.

Attention! This assignment is due on Wednesday, not Friday. It's also shorter than normal (but still contributes the usual amount to the course grade). There is another assignment coming on Wednesday.

(1) [5pt] Find two Laurent series expansions for $f(z) = \frac{1}{z^3-2z^4}$ centered at 0. Where each of them is valid? (*Hint:* For $|z| < 1/2$, use geometric series for $\frac{1}{1-2z}$. For $|z| > 1/2$, write $\frac{1}{1-2z} = \frac{-1}{2z}\frac{1}{1-\frac{1}{2z}}$ and use geometric series.)

(2) [5pt] Find the Laurent series centered at 0 for the following functions. (*Hint:* Use Taylor series for the involved trig and hyperbolic functions.)

(a) $\frac{\sinh z - \sin z}{z^8}$.
(b) $z^2 \cos \frac{1}{z}$.

(3) [6pt] Locate the zeros of the following functions and determine their order. (*Hint:* You can either go by the definition (the one about derivatives), or by in class statement that orders of zero at α add when functions are multiplied, or by finding Taylor series.)

(a) $(1+z^2)^3$.
(b) $1 - \cos z$.
(c) $\sin z^2$.
(d) $z^3 e^{z-1}$.

(4) [6pt] Locate the poles of the following functions and determine their order.

(a) $(z^2+1)^{-3}(z-1)^{-5}$.
(b) $z \cot z$.
(c) $z^{-5} \sin z$.
(d) $(z^2(1-\cos z))^{-1}$.

(5) [12pt] Locate the singularities of the following functions and determine their type: removable, pole of order n (find n), essential, non-isolated.

(a) $\tan^2 z$.
(b) $\frac{z}{\sin z}$.
(c) $\frac{\sin z}{z^2+z}$.
(d) $\frac{e^z-1}{z}$.
(e) $z^3 e^{\frac{1}{z}}$.
(f) $1/\cos \frac{1}{z}$.